Body Surface Area Formula Based on Geometric Means

LIPSCOMBE, Trevor*

Catholic University of America, 620 Michigan Avenue, NE 20064, Washington, DC, United States.

Article submited: 28.07.2020; accepted: 01.08.2020

Abstract

*We construct a new formula for body surface area based on geometric means. This we show, by means of Bland-Altman analysis and use of the Kolmogorov-Smirnov and Anderson-Darling tests, to be statistically equivalent to the arithmetic average of existing formulas. The result, the geometric mean formula (GMF) is the expression BSA = 0.00878108*W^ 0.434972*H^ 0.67844. We show also that body surfaces areas predicted by the GMF obey a lognormal distribution.*

Keywords: body surface area, anthropometry, biometry, statistical distributions

*Corresponding Author: Trevor Lipscombe; lipscombe@cua.edu

Introduction

Human body surface area is of great medical importance in a variety of clinical contexts. [1–4] especially when therapeutic agents are administered in doses that are surface-area dependent. Likewise, body temperature-regulation [5–7] depends on the physical processes of heat conduction, convection, radiation, and evaporation (sweating), all of which are surface-area dependent. As heat transfer across the skin is an energy-transfer process, body surface area also plays a vital role in weight regulation

Heat tolerance is a long-standing issue and recently has been discussed in relation to body surface area-to-mass ratio [8]. Prolonged physical activity generates higher core temperatures, which are surface-area dependent and, in some cases, require rapid treatment [9]. This rise in core temperature upon prolonged physical exertion occurs even in outer space. [10–11].

Using the appropriate expression for body surface area is key in any setting, but especially so in environments that are at the extreme in terms of temperatures or, indeed, of physical exertion, or when administering drugs whose dosages are area-dependent.

There are many formulas for body surface area, such as those proposed by Du Bois and Du Bois [12]; Mosteller [13]; Gehan and George [14], Shuter and Aslani [15], and Hayock et al. [16]. The most famous of these are due to Du Bois and Du Bois (one of the earliest) and that of Mosteller (one of the simplest, mathematically). Body surface area formulas are derived empirically. Namely, the body surface areas of a number of subjects are determined by a particular method (3D body scanning being the latest technology) and the subjects' weights and heights are also recorded. Then, by means of regression analysis, a best-fit formula is produced. In the case of Du Bois and Du Bois, the resulting expression is:

BSA = 0.007184 * W^0.425 * H^0.725

Here, BSA is the body surface area in meters, the height H is measured in centimeters and W the weight measured in kilograms. For Mosteller, it is:

BSA = (1/60) * W^0.5 * H^0.5

There is a bigger issue. There are large numbers of alternate empirical formulas, 20 of which are shown in Table One, in chronological order [17–34]. With so many choices, which should physicians and medical researchers use? Some researchers raise the alarm, showing the significant variation in the predictions of various formulas [35]. Others suggest that it does not particularly matter which one you use. [36] Part of the issue is that there is no theoretical formula established for body surface area with which to compare experimentally determined results [37].

In this article, we construct a formula for body surface area using the geometric mean. This formula we compare to the arithmetic mean for a group of 4,082 individuals, and show that, statistically speaking, each average is equally good. The geometric mean formula can easily be updated to accommodate new research.

Method

Each of the currently existing body-area formulas is a case in point. Namely, a sample of people was taken from the general population, and hence each of the formulas reflects the sample that was chosen. Here we combine them to generate a "best" formula for body surface area.

To do so, consider a set of N surface area formulas:

BSA(1) = k(1) * W^a(1) * H^b(1)

BSA(2) = k(2) * W^a(2) * H^b(2) BSA(N) = k(N) * W^a(N) * H^b(N)

Multiply these N formulas together to obtain:

BSA(1)*BSA(2)*…BSA(N) = k(1)*k(2)*…k(N) * W^[a(1)+a(2)+…a(N)] * H^[b(1) + b(2) + …b(N)]

Raise both sides to the power I/N. This gives:

[BSA(1)*BSA(2)*…BSA(N)]^(1/N)= [k(1)*k(2)*… k(N)]^(1/N)* W^[a(1)+a(2)+…a(N)]/N * H^[b(1) + b(2) + …b(N)]/N

However, if we take N numbers, multiply them together, and take the Nth root of the product, this is—by definition—the *geometric* mean, or the geometric average. Likewise, adding N numbers together and diving by N is the *arithmetic* mean or the arithmetic average.

This implies that if the geometric mean of the areas predicted by the 22 surface area formulas is a reliable estimator for the actual body surface area, then we can use the geometric mean formula (GMF), namely:

BSA = k * M^a * H^b

Here *k* is the *geometric* mean of k(1), k(2)…k(N); *a* is the *arithmetic* mean of the mass exponents a(1), a(2),… a(N)and *b* is the *arithmetic* mean of the height exponents b(1), b(2)…b(N).

The GMF assumes that the geometrical mean of the individual body surface areas is a reliable estimator for the actual body surface area. To test this hypothesis, we used the data from the ANSUR II survey of U.S. military personnel [38].. These report the heights and weights of 4,082 men. For each subject, we computed the body surface area predicted by each of the 20 formulas. These we added to form the arithmetical mean for each subject based on all these formulas. We then computed the geometrical average, by means of the GMF. Thus we have two sets of data for 4,082 men, which we seek to compare.

To do so, we used three nonparametric tests. Nonparametric tests are perhaps more appropriate as we do not wish to assume an underlying statistical distribution for surface area. First, we used Bland-Altman analysis [39]. This showed that the difference between the arithmetic mean and the geometric mean, BSA(arith) – BSA(geometric) had an arithmetic mean of 0.001572465193 m^2, so that areas predicted by the arithmetic mean of the various surface area formulas are, on average, about 16 cm^2 more than the areas predicted by the GMF. The standard deviation was $\sigma = 0.0002083404545$. Of the 4,082 subjects, 0 were below, and 224 were above the usual 95% confidence limits, namely, 1.96σ above or below the arithmetic mean. These outliers, then, constitute less than 5.5% of all subjects in the survey.

Next, we used the two-sample Kolmogorov-Smirnov test [40], by computing the empirical cumulative distribution functions for A(arith) and A(mean). The KS test statistic, D = 0.04294 was significantly below the critical value for a sample of this size, At a level of significance α <=0.001, and thus with a high degree of confidence, we believe that the two samples are from the same distribution.

Last, we used the most powerful non-parametric test in this context, the Anderson-Darling statistic [41]. Again, as AD = 0.101, with a confidence level α <0.01, we have no reason to believe the distributions of surface area for the geometrical average formula and those of the arithmetically averaged formula are different. From this we conclude that each approach is equally valid, and as the geometrical average is easy to compute, we can use that as a good measure for human body surface area.

Results and discussion

There are at least 20 body surface area formulas that have been determined empirically and which fit the exponent pattern. These are shown in Table One, for weight measured in kilograms and height in centimeters. (Mosteller's formula is excluded, as it was reported as being a simplification of Gehan and George's empirical formula. Mosteller reported his equation is a good fit, within 2%, but "is poorest in short, obese, adults." Likewise the formula of Yo, Lo, and Chiou as excluded as it assumed the same exponents as Mosteller, and used data only to obtain a best estimate for the multiplicative constant, k.)

As the GMF has been shown to be a good estimator for human body area, we computed the geometric mean of the individual constants k(1), k(2)…k(20) for the expressions in Table One, and also formed the arithmetic means of the exponents a(1), a(2)…a(N) and b(1), b(2)…b(N). This resulted in the GMF:

$$BSA = 0.00878108 * W^{0.434972} * H^{0.67844}$$

This is close to the simple, mathematically pleasing, and easy to remember expression,

$$BSA = 0.0088 * W^{(4/9)} H^{(2/3)}.$$

Whose exponents for weight and height are 2.18% above, and 1.74% below, the GMF values respectively. The constant in this simpler formula is 0.22% above the GMF value. This has the property associated with constant-density models of body surface area, in that it is then has the correct dimensional dependence on W and H, as its two exponents obey $3a + b = 2$.

Note that the exponents of the GMF are extremely close to those obtained by Shuter and Aslani:

$$BSA = 0.00949 W^{0.441} H^{0.665}.$$

with exponents 1.39% below and 1.98% below the GMF values. (The exponents for Fujimoto and Watanabe's formula are also close, but slightly worse, than Shuter and Aslani.) The constant is 8.08% above the GMF value.

As Table One shows, as more formulas were added to the medical literature, so the geometric mean for k, and the arithmetic means of a and b, change. The belief, then, is that as time passes and more formulas are derived based on more samples, the GMF will be robust and approach the "true" but unknown equation for the body surface area. The change in *a* from Dubois and Dubois' formula of 1916 (0.425) to the current value of 0.434972 is less than 2.35%; the change in *a* from Gehan and George in 1970 until now is a mere 1.84%. The change in *b* is more marked, 6.42% since 1916. However, from 1970 new data have affected the value of *b* far less, a drop of less than 3.85%. The change in k has been striking, increasing by over 22.2% of the 1916 value. Again, since 1970, the value has changed by 9.2%. The weight dependence is therefore fairly settled, and the exponent of height dependence is stabilizing. That said, since the advent of 3D scanning technology, the values of a average to 0.41538, b to 0.73496, and k to 0.006957. These are, quite remarkably, close to the values given by Du Bois and Du Bois.

The new surface area formula has some important features. First, it is easily to update. That is to say, whenever a new relationship between BSA, height, and weight is reported, this geometrical average formula can incorporate it immediately. Next, we would expect the numbers eventually to become almost constant as new formulas are added, so that we will have honed in on the "true" relationship between the triad of BSA, weight, and height. Last, it is a "safe" formula. Namely, as it is an average, it will never yield the highest, nor the lowest, estimate of body surface area, compared to any other formula. This can be vitally important when administering a drug whose dose is determined by body surface area, for which one does not wish the peak to be toxic or the trough not to be therapeutic.

There is evidence that both weight and height obey (correlated) lognormal distributions. This suggests that BSA should follow a lognormal distribution as well. As a consequence, we tested the natural logarithm of the BSA areas as predicted by the GMF for the 4,082 subjects in the survey. At a level of significance $\alpha = 0.01$. BSA (as given by the GMF) obeys a lognormal distribution, as predicted by a Shapiro-Wilks test, with W = 0.999388. The mean was 0.706184, corresponding to a mean surface area of 2.026244 m^2, and almost exactly coinciding with the median, 0.7067159387. The sample standard deviation was 0.0872524. The skewness was -0.0241070, consistent with an almost symmetrical distribution, and the excess kurtosis was 0.113959, so that the data is mesokurtic, as is the normal distribution. Given the non-parametric tests carried out on the GMF and the arithmetic mean of the existing formulas, the analysis strongly suggests that body surfaces areas follow a lognormal distribution.

Conclusions

A new expression for body surface area, the GMF, has been derived. It is the geometric mean of the pre-existing formulas. We have shown it is a reliable estimator of body surface areas. It is adaptable, and can incorporate any new body surface area formulas that are proposed. Because of the nature of an average, we believe that as new empirically determined expressions for BSA are reported, the improved GMF that results will ever-more-closely approximate the actual, and possibly unknowable, formula for body surface area.

Table 1. Formulas for BSA showing cumulative averages

Name	k	a	b	mean k	mean a	mean b
Dubois (1916)	0.007184	0.425	0.725	0.007184	0.425	0.725
Faber and Melcher (1921)	0.00785	0.425	0.725	0.00750962	0.425	0.725
Takahira (1925)	0.007246	0.425	0.725	0.025794458	0.425	0.725
Brody (1945)	0.02411	0.53	0.4	0.000426056	0.45125	0.64375
Banerjee and Sen (1955)	0.007466	0.425	0.725	0.025448405	0.446	0.66
Choi (1956)	0.005902	0.407	0.776	0.009125322	0.4395	0.679333
Mehra (1958)	0.01131	0.4092	0.6468	0.008574575	0.435171	0.674686
Banerjee and Bhattarcharya (1961)	0.007	0.425	0.725	0.008876543	0.4339	0.680975
Fujimoto and Watanabe (1961)	0.008883	0.444	0.663	0.008645363	0.435022	0.678978
Gehan and George (1970)	0.0235	0.51456	0.42246	0.008668838	0.442976	0.653326
Haycock et al. (1978)	0.024265	0.5378	0.3964	0.009491486	0.451596	0.629969
Anderson (1985)	0.0239	0.517	0.417	0.010263719	0.457047	0.612222
Nwoye (1989)	0.001315	0.262	1.2139	0.010953242	0.442043	0.658505
Shuter and Aslani (2000)	0.00949	0.441	0.655	0.00941422	0.441969	0.658254
Tikuisis (2001)	0.01281	0.44	0.6	0.009419253	0.441837	0.654371
Nwoye and Al-Sheri (2003)	0.02036	0.427	0.516	0.009602012	0.44091	0.645723
Yu, Lin, Yang	0.00714	0.404	0.7437	0.010036058	0.438739	0.651486
Schlich et al.(2010)	0.000579	0.38	1.24	0.009848001	0.435476	0.684181
Kuehnapfel et al. (2017)	0.0151	0.4259	0.5751	0.008483887	0.434972	0.67844

Resumo

Ni konstruas novan formulon por areo de korpa surfaco (angle, BSA) surbaze de geometriaj mezumoj. Tion ni montras, per Bland-Altman-analizo kaj uzo de la testoj de Kolmogorov-Smirnov kaj Anderson-Darling, esti statistike ekvivalenta al la aritmetika mezumo de ekzistantaj formuloj. La rezulto (angle, GMF) estas la esprimo BSA = 0.0108651 W ^ 0.443446 H ^ 0.647407. Ni montras ankaŭ, per uzo de la testo statistika de Shapiro-Wilks, ke BSA-o obeas lognorman distribuon.

References

1. Sawrey EL, Subramanian MW, Ramirez KA, Snyder BS, Logston BB, Russell GB. Use of Body Surface Area for Dosing of Vancomycin. J. Pediatr. Pharmacol Ther. 2019; Jul-Aug; 24(4):296–303. doi: 10.5863/1551–6776–24.4.296

2. Brooksbank JA, Greene SJ, DeWald TA, Mentz RJ. Body surface area and medication dosing in patients with heart failure with reduced ejection fraction. Trends Cardiovasc Med. 2020; Jan 15. pii: S1050–1738(20)30005–0. doi: 10.1016/j.tcm. 2019. 12.011

3. Basu B, Bhattacharyya S, Barua S, Naskar A, Roy B. Efficacy of body weight vs body surface area-based prednisolone regimen in nephrotic syndrome. Clin Exp Nephrol. 2020; Mar 22. doi: 10.1007/s10157–020–01875-y

4. Palmu, S, Rehunan, S, Kautiainen, H, Eriksson, JG, Korhonen, PE. Body surface area and glucose intolerance: The Smaller the Person the Greater the Two-Hour Plasma Glucose. Diabetes Res. Clin. Pract. 2019; 2019 Nov;157:107877. doi: 10.1016/j.diabres.2019. 107877

5. Yaglou CP, Minard, D. Control of heat casualties at military training centers AMA Arch. Ind. Health 1957; 16(4) 302–16.

6. Ellis, FP. Prevention of heat incapacitation in the Armed Forces. Military Medicine 1955; 116(5): 323–9.

7. Ellis, FP. Heat illness J. R. Nav. Med. Serv. 1958; 44(4) 236–48.

8. Lisman P, Kazman JB, O'Connor FG, Heled Y, Deuster PA.. Heat tolerance testing: association between heat intolerance and anthropomorphic and fitness measurements. Military Medicine 2014; 179 (11) 1339–1346. doi: 10.7205/MILMED-D-14–00169

9. Sagui E, Beighau S, Jouvion A, Trichereau J, Cornet D, Berthelot RC, Canini F, and Grelot L, et al. Thermoregulatory response to exercise after exertional heat stroke. Military Medicine 2017; 182(7) e1842-e1850 doi: 10.7205/MILMED-D-16–00251

10. Stahn AC, Werner A, Opatz O, Maggioni MA, Steinach M, Weller von Ahlefeld V, Moore A, Crucian BE, Smith SM, Zwart SR, Schlabs T, Mendt S, Trippel T, Koralewski E, Koch J, Choukèr A, Reitz G, Shang P, Röcker L, Kirsch KA, and Gunga, H-C. Increased body core temperature in astronauts during long-duration space missions. Sci. Rep. 2017; 7(1) 16180 doi: 10.1038/s41598–017–15560-w

11. Pisacane VL, Kuznetz LH, Logan JS, Clark JB, Wissler, EH. Thermoregulatory models of space shuttle and space station activities. Aviat Space Environ Med. 2007; Apr;78(4 Suppl):A48–55.

12. Du Bois D and Du Bois, EF. A formula to estimate the approximate surface area if height and weight be known. Archives of Internal Medicine. 1916; 17 (6): 863–71.

13. Mosteller RD. Simplified calculation of body-surface area. N Engl. J. Med. 1987; 317 (17): 1098. See the subsequent letter exchange

with Lam TK and Leung DTY More on Simplified Calculation of Body Surface Area N. Engl. J. Med. 1988; 318 (17): 1130. Doi 1056| NEJM 198804283181717.

14. Gehan EA, and George SL. Cancer Chemother. Rep. 1970; 54:225–235.

15. Shuter B and Aslani A. Body surface area: Du Bois and Du Bois revisited. European Journal of Applied Physiology.2000; 82 (3): 250–254.

16. Haycock GB, Schwart z GJ, and Wisotsky DH. Geometric Method for measuring body surface area: A height-weight formula valid for infants, children, and adults J. Paediatr. 1978; 93(1) 62–66.

17. Faber HK, and Melcher M S. A modification of the Du Bois height-weight formula for surface areas of newborn infants. Exp. Biol. Med. 1921; 19, 53–54.

18. Brody S. Bioenergetics and growth. New York: Reinhold Publishing Company; 1945.

19. Takahira H. *Metabolism of the Japanese.* No. 1 in Imperial Government Institute of Nutrition. Report (The Institute, 1925).

20. Banerjee S and Sen R. Determination of the Surface Area of the Body of Indians. J. Appl. Physiol. 1955;7, 585–588.

21. Choi W.R. *The body surface area of Koreans.* Ph.D. thesis, Seoul National University (1956).

22. Mehra N C. Body Surface Area of Indians. J. Appl. Physiol. 1958; 12, 34–36.

23. Banerjee S, and Bhattacharya AK. Determination of body surface area in Indian Hindu children. J. Appl. Physiol. 1961; 16, 969–970.

24. Fujimoto S and Watanabe T. Studies on the body surface area of Japanese. Acta Med Nagasaki. 1969; 14(1):1–13.

25. Anderson E. Development of statistical distributions or ranges of standard factors used in exposure assessments. Final report. Office of Health and Environmental Assessment, Office of Research and Development. Washington, D.C: US Environmental Protection Agency; 1985.

26. Nwoye L. O. Body Surface Area of Africans: A Study Based on Direct Measurements of Nigerian Males. Hum. Biol. 1989; 61, 439–457.

27. Tikuisis P, Meunier PJ, and ubenville CE. Human body surface area: measurement and prediction using three dimensional body scans. Eur J Appl Physiol. 2001; 85(3–4):264–271. doi: 10.1007/s004210100484

28. Nwoye LO. & Al-Shehri MA. A formula for the estimation of the body surface area of Saudi male adults. Saudi Med. J. 2003; 24, 1341–1346.

29. Yu CY, Lin CH, and Yang YH. Human body surface area database and estimation formula. Burns. 2010; Aug; 36(5):616–29. doi: 10.1016/j.burns.2009.05.013

30. Schlich E, Schumm M, and Schlich M. 3D-Body-Scan als anthropometrisches Verfahren zur Bestimmung der spezifischen Körperoberfläche. Ernährungs Umschau 2010; 4, 178–183.

31. Kuehnapfel A, Ahnert P, Loeffler M,and Scholz, M. Body surface assessment with 3D laser-based anthropometry: reliability, validation, and improvement of empirical surface formulae Eur J Appl Physiol. 2017; 117(2): 371–380.doi: 10.1007/ s00421–016–3525–5

32. Redlarski, G, Palkowski, A, and Krawczuk M. Body surface area formulae: An Alarming Ambiguity, Sci. Rep. 2016; June 21; 6:27966. doi: 10.1038/srep27966

33. Faisal, W, Tang HM, Tiley, S, and Kukard

C. Not All Body Surface Area Formulas are the Same, but Does it Matter. J. Glob. Oncol. 2016; Jul 20;2(6):436–437. doi: 10.1200/JGO. 2016. 005876

34. There is, though, Burton RF. Estimating body surface area from mass and height: theory and the formula of Du Bois and Du Bois. Ann. Hum. Biol. 2008; Mar-Apr; 35(2):170–84. doi: 10.1080/0301446 080190 8439. See also Wang J and Hihara E. Human body surface area: a theoretical approach. Eur J Appl Physiol. 2004; 91(4):425–428. doi: 10.1007/s00421–003–1011–3. In a similar vein, the following article assumes Mosteller's power law and then estimates a best fit for the constant. Yu C-Y, Lin C.-H, and Yang Y-H. Human body surface area database and estimation formula. Burns 2010; 36, 616–629 doi: 10.1016/ j.burns.2009. 05.013.

35. The data is freely downloadable from https://www.openlab.psu.edu/ansur2/

36. See, for example, Giavarina D. Understanding Bland-Altman Analysis. Biochem Med (Zagreb). 2015; Jun; 25(2): 141–151. doi: 10.11613/BM.2015.015

37. See, for example,GolverT and Mitchell K, Introduction to Biostatistics, Third Edition, 2016; Longrove, IL: Waveland pp. 376–381.

38. Anderson TW and Darling, DA. A Test of Goodness of Fit. Journal of the American Statistical Association 1954; 49(268): 765–769.